

# Warped de Sitter compactifications

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**ABSTRACT:** We show that the warped de Sitter compactifications are possible under certain conditions in  $D$ -dimensional gravitational theory coupled to a dilaton, a form field strength, and a cosmological constant. We find that the solutions of field equations give de Sitter spacetime with the warped structure, and discuss cosmological models directly obtained from these solutions. We also construct a cosmological model in the lower-dimensional effective theory. If there is a field strength having non-vanishing components along the internal space, the moduli can be fixed at the minimum of the effective potential where a de Sitter vacuum can be obtained.

**KEYWORDS:** Flux compactifications, Classical Theories of Gravity.

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## 1. Introduction

de Sitter compactification of higher-dimensional theory is an important cosmological issue. Such a solution has been explored from several points of view because this provides a fairly direct explanation of the accelerating expansion of four-dimensional universe. The inflation and accelerating expansion [1, 2, 3, 4, 5, 6] with warped compactifications can be constructed in a variety of ways from higher-dimensional cosmology or string theory [7, 8, 9]. Actually, the proposal for a physical construction of de Sitter compactification has been made recently [9, 10, 11, 12, 13, 14, 15, 16]. In the gravity theory, an initial clue of the de Sitter compactification was that the hyperbolic space associated to a higher-dimensional theory can be regarded as internal space [17, 18, 19, 20, 21, 22, 23, 24] (See also [25, 26, 27, 28, 29]). It is not hard to see why a de Sitter compactification give an the hyperbolic space that can be derived from higher-dimensional Einstein equations. We can see, roughly speaking, that a curvature of the de Sitter space can be compensated by that of the internal space. Then, the curvature of the internal space has the negative sign.

The warped de Sitter compactification shed much light on whether there was the exact solution of field equations because the solution such as D-branes or M-branes in the supergravity can be embedded in warped compactification. Thus the cosmological solutions of the  $p$ -brane system have been discussed in ten- or

eleven-dimensional supergravity theory as the examples of warped compactifications [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41]. These solutions are very efficient for constructing the cosmological models, and showing that they indeed are compactifications with warped structure [35, 36, 42]. However, such constructions cannot make the accelerating expansion of our four-dimensional universe manifest yet [43, 44, 45, 46].

A road to obtain the warped de Sitter compactification has appeared recently in a study of the higher-dimensional pure gravity theory in [47]. It was shown that warped structure of the spacetime realizes a de Sitter universe, but the pay is that one of the internal space dimensions has to have an infinite volume rather than small and finite one. For such a class of solutions, a way of the construction of a cosmological model is to insert a brane world boundary in the noncompact direction whose world volume contains all the remaining compact directions of the internal space which is discussed more explicitly in [47] (see also [32, 43, 45]). It may be regarded as a generalization of the five-dimensional brane world models (see e.g., [48]). The details of the brane world models obtained from our solutions are argued in [49].

In the first part of the paper, we will focus on the derivation of the new warped de Sitter solutions obtained as the generalization of Ref. [47]. We begin with the pure gravity theory in  $D$  dimensions, and re-examine possible generalizations of the de Sitter compactification [47], with the internal space of positive and negative curvature. This gives the logically simple treatment of this topic and the clearest explanation of warped compactifications. In the case with a positively curved internal space, we will show a simpler expression of the solution [47]. We then discuss the warped de Sitter compactification with several matter fields, in particular a scalar and gauge fields in higher-dimensional gravity theory because of the following two reasons.

One is that in general the solutions with matter fields are not given simply via the trivial extension of the pure gravity solutions [47]. How the matter fields backreact on the pure gravity solution is a highly nontrivial question. Even though the spacetime structures with and without matter have similarities in some regions, in other regions they lead to different pictures. An instructive example is the case of static and spherically symmetric black holes in four dimensions, namely Schwarzschild black hole (without the gauge field) and Reissner-Nordström black hole (with the gauge field). Although both solutions have asymptotically flat infinities if there is no cosmological constant, the inner structure of these black holes can be quite different. A similar but more relevant example to us is the case of the time-dependent D3-brane solutions in the ten-dimensional type IIB supergravity with the trivial dilaton [30, 33]. In the presence of the 5-form field strength, the asymptotic spacetime structure behaves as a Kasner spacetime and contains the horizons of black 3-branes, while in the absence of it the whole spacetime structure is exactly described by a Kasner spacetime with no horizons. Therefore, whether (and how) the matter field, in particular,

the gauge field gives some differences in the spacetime structure for the warped de Sitter solutions should be carefully studied. Moreover, a variety of phenomenologically interesting objects in the higher-dimensional theories arise in the presence of matter fields. For example, D-branes in string theory arise if the gravity is coupled to several combinations of scalars and forms.

The other reason is to achieve the stabilization of the internal space modulus by changing properties of the internal space, the matter fields or other details. Such an analysis can be particularly meaningful by adding gauge fields [50], and cannot be achieved in the previous papers [47] without the gauge field. The importance of the gauge field can be seen at the level of the cosmological model, for example to obtain the well-behaved effective four-dimensional Planck mass in the limit of the vanishing expansion rate [49]. Along the way, we will clarify some physical properties of warped de Sitter compactifications.

In the second part of this paper, we will investigate a stabilization mechanism of the internal space via some kinds of matter fields in the  $D$ -dimensional theory, and present another way to construct a cosmological model adding matter fields after integrating over the compact directions and involving the noncompact direction in the higher-dimensional solution into our Universe. We consider the matter fields with a cosmological constant in order to stabilize the scale of internal space. Many works suggest that the field strength might provide a physical mechanism which is capable of accounting for the extreme smallness of the extra dimensions [51, 52, 53, 54].

The present paper is constructed as follows. We give the warped de Sitter compactifications in  $D$ -dimensional pure gravity theory in Sec. 2. As the starting point, we will investigate the  $D$ -dimensional solutions in the pure gravity which are given in terms of the warped product of the  $(n+1)$ -dimensional external spacetime  $M$  and the  $(D-n-1)$ -dimensional internal space  $Z$ . A de Sitter compactification is obtained if we specialize to the case that the  $(n+1)$ -dimensional spacetime  $M$  is given by the product of  $M = X \times \mathbb{R}$  with the warped structure, where  $X$  is being an  $n$ -dimensional de Sitter spacetime. We then present the solutions to be warped compactification with several matter fields, and this will also lead to  $n$ -dimensional de Sitter space in Sec. 3. We then briefly discuss the brane world models where the  $n$ -dimensional  $X$  space becomes our de Sitter Universe, following [49]. In Sec. 4, we also discuss the stabilization of moduli degrees of freedom in the lower-dimensional effective theories for the warped compactification. We present a construction of the cosmological model in which a cosmological  $(n+1)$ -dimensional spacetime  $\bar{M}$  is obtained from a compactification of the  $D$ -dimensional theory onto a  $(D-n-1)$ -dimensional Einstein space. Here  $\bar{M}$  is conformally related to the  $(n+1)$ -dimensional metric  $M$ . After the dimensional reduction, the moduli degrees of freedom are coupled to the matter fields in the external spacetime. Upon reducing on the internal space, the  $(n+1)$ -dimensional effective action would lead to the moduli potential which could have a local minimum of the potential, where the Einstein frame manifold  $\bar{M}$  becomes

our  $(n + 1)$ -dimensional de Sitter universe. Sec. 5 is devoted to giving concluding remarks.

## 2. De Sitter spacetime in $D$ -dimensional warped compactifications

In this section, we construct the cosmological warped compactification which gives the accelerating expansion of the universe in the pure gravity.

We take the following ansatz for the  $D$ -dimensional metric

$$ds^2 = e^{2A_1(y)} \left[ e^{2A_0(x)} q_{\mu\nu}(X) dx^\mu dx^\nu + u_{ij}(Y) dy^i dy^j \right], \quad (2.1)$$

where  $q_{\mu\nu}$  is a  $n$ -dimensional metric which depends only on the  $n$ -dimensional coordinates  $x^\mu$ , and  $u_{ij}$  is the  $(D - n)$ -dimensional metric which depends only on the  $(D - n)$ -dimensional coordinates  $y^i$ .

In terms of the  $D$ -dimensional metric (2.1), the Einstein equations are written as

$$R_{\mu\nu}(X) - (n - 2)D_\mu D_\nu A_0 + (n - 2)\partial_\mu A_0 \partial_\nu A_0 - q_{\mu\nu} [\Delta_X A_0 + (n - 2)q^{\rho\sigma} \partial_\rho A_0 \partial_\sigma A_0] - e^{2A_0} q_{\mu\nu} [\Delta_Y A_1 + (D - 2)u^{kl} \partial_k A_1 \partial_l A_1] = 0, \quad (2.2a)$$

$$R_{ij}(Y) - (D - 2)D_i D_j A_1 + (D - 2)\partial_i A_1 \partial_j A_1 - u_{ij} [\Delta_Y A_1 + (D - 2)u^{kl} \partial_k A_1 \partial_l A_1] = 0, \quad (2.2b)$$

where  $D_\mu$ ,  $D_i$  are the covariant derivatives with respect to the metric  $q_{\mu\nu}$ ,  $u_{ij}$ , and  $\Delta_X$  and  $\Delta_Y$  are the Laplace operators on the space of  $X$  and the space  $Y$ , and  $R_{\mu\nu}(X)$  and  $R_{ij}(Y)$  are the Ricci tensors of the metrics  $q_{\mu\nu}$  and  $u_{ij}$ , respectively.

In the following, we consider the warped de Sitter compactifications with three kinds of the internal manifold.

### 2.1 Internal space with positive curvature

Let us consider the vacuum solution with the internal space which has the positive curvature [47]. We see the solution whose  $(D - n)$ -dimensional metric has the form

$$u_{ij}(Y) dy^i dy^j = \rho^2 [G(y) dy^2 + E \gamma_{ab}(Z) dz^a dz^b], \quad (2.3)$$

where  $G(y)$  and  $E$  are given by [47]

$$E = \frac{1}{3}(D - n - 2), \quad G(y) = \frac{1}{12}(D - n + 2) (\partial_y A_1)^2. \quad (2.4)$$

Though this  $(D - n)$ -dimensional metric has been already present in [47], we discuss the dynamical solution beyond what has already been cited. In terms of the metric

of the internal space Eq. (2.4), the Einstein equations Eq. (2.2) are given by

$$R_{\mu\nu}(X) - (n-2)D_\mu D_\nu A_0 + (n-2)\partial_\mu A_0 \partial_\nu A_0 - q_{\mu\nu} [\triangle_X A_0 + (n-2)q^{\rho\sigma}\partial_\rho A_0 \partial_\sigma A_0] - c_v e^{2A_0} q_{\mu\nu} = 0, \quad (2.5)$$

$$R_{ab}(Z) - 4(D-n-2)(D-n+2)^{-1}(D-2)\gamma_{ab}(Z) = 0, \quad (2.6)$$

where  $R_{ab}(Z)$  is the Ricci tensor with respect to the metric  $\gamma_{ab}(Z)$ , and  $c_v$  is defined by

$$c_v \equiv \frac{12(D-2)}{\rho^2(D-n+2)}. \quad (2.7)$$

Suppose that the function  $A_0$  is such that  $A_0 = A_0(t)$ . We set the  $n$ -dimensional spacetime metric  $q_{\mu\nu}$

$$q_{\mu\nu}(X)dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{mn}dx^m dx^n, \quad (2.8)$$

where  $\delta_{mn}$  is the metric of  $(n-1)$ -dimensional Euclidean space. Form the Eq. (2.5), we find

$$(n-1) \left[ \left( \frac{\dot{a}}{a} \right) + \left( \frac{\dot{a}}{a} \right)^2 + \ddot{A}_0 + \dot{A}_0^2 \right] - c_v e^{2A_0} = 0, \quad (2.9a)$$

$$\left[ \left( \frac{\dot{a}}{a} \right) + (n-1) \left( \frac{\dot{a}}{a} \right)^2 + \ddot{A}_0 + (2n-3)\frac{\dot{a}}{a}\dot{A}_0 + (n-2)\dot{A}_0^2 - c_v e^{2A_0} \right] q_{mn} = 0, \quad (2.9b)$$

where  $\dot{\phantom{x}}$  denotes the ordinary derivative with respect to the coordinate  $t$ . We assume that the functions  $A_0(t)$  and  $a(t)$  are given by

$$\dot{A}_0 = \xi_1 e^{A_0}, \quad \frac{\dot{a}}{a} = \xi_2 e^{A_0}, \quad (2.10)$$

where  $\xi_1$  and  $\xi_2$  are constants. Thus, the Einstein equations Eq. (2.9) are reduced to

$$[(n-1)(\xi_1 + \xi_2)^2 - c_v] e^{2A_0} q_{\mu\nu} = 0. \quad (2.11)$$

In order to satisfy Eq. (2.11), the constant  $c$  obeys

$$c_v = (n-1)(\xi_1 + \xi_2)^2. \quad (2.12)$$

Form the Eq. (2.10), we find

$$A_0(t) = -\ln [-(\xi_1 t + \xi_3)], \quad a(t) = a_0 (\xi_1 t + \xi_3)^{-\xi_2/\xi_1}, \quad (2.13)$$

where  $\xi_3$  and  $a_0$  are constants. Then, the metric of  $D$ -dimensional spacetime can be written as

$$ds^2 = e^{2A_1(y)} [-d\tau^2 + a_0^2 e^{2H\tau} \delta_{mn} dx^m dx^n + u_{ij}(Y) dy^i dy^j], \quad (2.14)$$

where the cosmic time  $\tau$  and the Hubble parameter  $H$  are given by

$$\tau = \pm \frac{1}{\xi_1} \ln [ - (\xi_1 t + \xi_3) ] , \quad (2.15a)$$

$$H \equiv \frac{d \ln a}{d\tau} = - (\xi_1 + \xi_2) = \pm \sqrt{\frac{c_v}{n-1}} . \quad (2.15b)$$

If we consider the case

$$\xi_1 + \xi_2 = - \sqrt{\frac{c_v}{n-1}} , \quad (2.16)$$

the solution leads to an accelerating expansion of the  $n$ -dimensional spacetime.

In terms of the ansatz Eq. (2.10), the condition  $A_0 = 0$  corresponds to  $\xi_1 = 0$ . Then, the solution for  $a(t)$  is given by

$$a(t) = \exp \left( \pm \sqrt{\frac{c_v}{n-1}} t \right) , \quad (2.17)$$

where we used the definition Eq. (2.7).

Clearly, in this case, we can choose  $A_0(t) = 0$  without loss of generality because  $A_0(t)$  is the degree of freedom corresponding to the scale of time. Up to an inessential scaling, the solution of the Einstein equations Eq. (2.2) can be written by

$$ds^2 = e^{2A_1(y)} \left[ -dt^2 + e^{2Ht} \delta_{mn} dx^m dx^n + u_{ij}(Y) dy^i dy^j \right] , \quad (2.18)$$

where we used the Hubble parameter

$$H^2 = \frac{c_v}{n-1} . \quad (2.19)$$

As the Hubble parameter  $H$  is proportional to the constant  $c_v$ , the internal manifold becomes flat space in the limit  $H \rightarrow 0$ .

The warp factor  $A_1(y)$  is determined by integrating Eq. (2.4) after specifying the function  $G(y)$ . Choosing the function  $G(y) = \rho^{-2}$ , it can be described in a much simpler way than in [47]. Then, integrating Eq. (2.4) the warped factor becomes

$$A_1(y) = -\frac{2}{\rho} \sqrt{\frac{3}{D-n+2}} (y - y_0) = -\sqrt{\frac{n-1}{D-2}} H (y - y_0) , \quad (2.20)$$

where we have used Eq. (2.7) and Eq. (2.19).

## 2.2 Internal space with negative curvature

Next we consider a solution which has the internal space with negative curvature. We assume that the  $(D-n)$ -dimensional metric takes the form

$$u_{ij}(Y) dy^i dy^j = dy^2 + e^{2\sigma y} \delta_{ab} dz^a dz^b , \quad (2.21)$$

where  $\sigma$  is constant and  $\delta_{ab}$  denotes the metric of the  $(D - n - 1)$ -dimensional torus. The Einstein equations Eq. (2.2b) then reduce to

$$(D - 2)\partial_y^2 A_1 + (D - n - 1)\sigma(\partial_y A_1 + \sigma) = 0, \quad (2.22a)$$

$$[\partial_y^2 A_1 + \{(D - n - 1)\sigma + (D - 2)\partial_y A_1\}(\partial_y A_1 + \sigma)]u_{ab} = 0. \quad (2.22b)$$

These equations are satisfied if the function  $A_1$  is given by

$$A_1 = -\sigma(y - y_0), \quad (2.23)$$

where  $y_0$  is constant. In terms of Eq. (2.23), Eq. (2.2a) leads to

$$R_{\mu\nu}(X) - (n - 2)D_\mu D_\nu A_0 + (n - 2)\partial_\mu A_0 \partial_\nu A_0 - q_{\mu\nu}[\Delta_X A_0 + (n - 2)q^{\rho\sigma}\partial_\rho A_0 \partial_\sigma A_0] - \chi e^{2A_0} q_{\mu\nu} = 0, \quad (2.24)$$

where the constant  $\chi$  is defined by

$$\chi \equiv \Delta_Y A_1 + (D - 2)u^{kl}\partial_k A_1 \partial_l A_1 = (n - 1)\sigma^2. \quad (2.25)$$

We set the function  $A_0 = 0$  from the beginning and choose the  $n$ -dimensional space-time metric  $q_{\mu\nu}$  as follows:

$$q_{\mu\nu}(X)dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{mn}dx^m dx^n, \quad (2.26)$$

where  $\delta_{mn}$  is the metric of  $(n - 1)$ -dimensional Euclidean space.

Upon setting (2.26), the Einstein equations (2.2a) are given by

$$\left[ (n - 1) \left\{ \left( \frac{\dot{a}}{a} \right) + \left( \frac{\dot{a}}{a} \right)^2 \right\} - \chi \right] q_{tt} = 0, \quad (2.27a)$$

$$\left[ \left( \frac{\dot{a}}{a} \right) + (n - 1) \left( \frac{\dot{a}}{a} \right)^2 - \chi \right] q_{mn} = 0. \quad (2.27b)$$

They give  $\left( \frac{\dot{a}}{a} \right) = 0$ , and the Hubble parameter  $H$  are defined as

$$H \equiv \frac{d \ln a}{dt} = \pm \sqrt{\frac{\chi}{n - 1}} = \pm \sigma. \quad (2.28)$$

We find the  $D$ -dimensional metric to be

$$ds^2 = e^{-2\sigma(y - y_0)} (-dt^2 + e^{\pm 2\sigma t} \delta_{mn} dx^m dx^n + dy^2) + e^{2\sigma y_0} \delta_{ab} dz^a dz^b, \quad (2.29)$$

where the metric  $\delta_{ab}$  denotes the  $(D - n - 1)$ -dimensional torus. The metric Eq. (2.29) takes the same form as the  $D$ -dimensional Minkowski spacetime. However, our solution is the product of the  $n$ -dimensional de Sitter spacetime and the internal space of  $\mathbb{R} \times T^{(D - n - 1)}$ , which is different from the trivial Minkowski spacetime. Since the internal space except for the  $\mathbb{R}$  direction has a finite volume, this solution represents a de Sitter warped compactification.



### 3. Compactifications with matter fields

In this section, we discuss the warped de Sitter compactification which includes several fields and the cosmological constant.

#### 3.1 Compactifications with scalar field

Let us first consider the warped de Sitter compactification which includes the metric  $g_{MN}$ , the scalar fields  $\phi$ ,  $\varphi$  the cosmological constant  $\Lambda$ , to see how the ansatz strictly restricts the possibility of de Sitter compactifications [16]. The action we consider in the Einstein frame is given by

$$S = \frac{1}{2\kappa^2} \int \left[ (R - 2e^{\alpha_\phi \phi} \Lambda) * \mathbf{1} - \frac{1}{2} d\phi \wedge * d\phi - \frac{1}{2} d\varphi \wedge * d\varphi \right], \quad (3.1)$$

where  $\kappa^2$  is the  $D$ -dimensional gravitational constant,  $*$  is the Hodge operator in the  $D$ -dimensional spacetime, and  $\alpha_\phi$  is defined by

$$\alpha_\phi = \sqrt{\frac{2}{D-2} + c}, \quad (3.2)$$

with the constant  $c$ .

After varying the action Eq. (3.1) with respect to the metric and the dilaton, we obtain the field equations

$$R_{MN} = \frac{1}{2} \partial_M \varphi \partial_N \varphi + \frac{1}{2} \partial_M \phi \partial_N \phi + \frac{2}{D-2} e^{\alpha_\phi \phi} \Lambda g_{MN}, \quad (3.3a)$$

$$d * d\phi - 2\alpha_\phi e^{\alpha_\phi \phi} \Lambda * \mathbf{1} = 0, \quad (3.3b)$$

$$d * d\varphi = 0. \quad (3.3c)$$

To solve the field equations, we assume that the  $D$ -dimensional metric takes the form

$$ds^2 = e^{2A_1(y)} \left[ e^{2A_0(x)} q_{\mu\nu}(X) dx^\mu dx^\nu + u_{ij}(Y) dy^i dy^j \right], \quad (3.4)$$

where  $q_{\mu\nu}$  is a  $n$ -dimensional metric which depends only on the  $n$ -dimensional coordinates  $x^\mu$ , and  $u_{ij}$  is the  $(D-n)$ -dimensional metric which depends only on the  $(D-n)$ -dimensional coordinates  $y^i$ . Furthermore, we assume that the scalar fields  $\phi$  and  $\varphi$  are given by

$$\phi = -\frac{2}{\alpha_\phi} A_1(y), \quad \varphi = \varphi_c A_0(x), \quad (3.5)$$

where  $\varphi_c$  is a constant.

Let us first consider the Einstein equations Eq. (3.3a). Using the assumptions Eq. (3.4) and Eq. (3.5), the Einstein equations are given by

$$R_{\mu\nu}(X) - (n-2)D_\mu D_\nu A_0 + \left(n-2 - \frac{\varphi_c^2}{2}\right) \partial_\mu A_0 \partial_\nu A_0 - q_{\mu\nu} [\Delta_X A_0 + (n-2)q^{\rho\sigma} \partial_\rho A_0 \partial_\sigma A_0 + e^{2A_0} \left\{ \Delta_Y A_1 + (D-2)u^{kl} \partial_k A_1 \partial_l A_1 + \frac{2}{D-2} \Lambda \right\}] = 0, \quad (3.6a)$$

$$R_{ij}(Y) - (D-2)D_i D_j A_1 + \left(D-2 - \frac{2}{\alpha_\phi^2}\right) \partial_i A_1 \partial_j A_1 - u_{ij} \left[ \Delta_Y A_1 + (D-2)u^{kl} \partial_k A_1 \partial_l A_1 + \frac{2}{D-2} \Lambda \right] = 0, \quad (3.6b)$$

where  $D_\mu$ ,  $D_i$  are the covariant derivatives with respect to the metrics  $q_{\mu\nu}(X)$ ,  $u_{ij}(Y)$ , and  $\Delta_X$ ,  $\Delta_Y$  are the Laplace operators on the space of  $X$  and the space  $Y$ , and  $R_{\mu\nu}(X)$ ,  $R_{ij}(Y)$  are the Ricci tensors of the metrics  $q_{\mu\nu}(X)$ ,  $u_{ij}(Y)$ , respectively.

Let us next consider the scalar field equation. Substituting Eq. (3.5) into Eqs. (3.3b) and (3.3c), we obtain

$$\frac{2}{\alpha_\phi} e^{-2A_1} [\Delta_Y A_1 + (D-2)u^{kl} \partial_k A_1 \partial_l A_1 + \alpha_\phi^2 \Lambda] = 0, \quad (3.7a)$$

$$\varphi_c e^{-2A_0} [\Delta_X A_0 + (n-2)q^{\rho\sigma} \partial_\rho A_0 \partial_\sigma A_0] = 0. \quad (3.7b)$$

Combining Eq. (3.6) with Eq. (3.7), we get

$$R_{\mu\nu}(X) - (n-2)D_\mu D_\nu A_0 + \left(n-2 - \frac{\varphi_c^2}{2}\right) \partial_\mu A_0 \partial_\nu A_0 - [\Delta_X A_0 + (n-2)q^{\rho\sigma} \partial_\rho A_0 \partial_\sigma A_0 - c\Lambda e^{2A_0}] q_{\mu\nu} = 0, \quad (3.8a)$$

$$R_{ij}(Y) - (D-2)D_i D_j A_1 + \left(D-2 - \frac{2}{\alpha_\phi^2}\right) \partial_i A_1 \partial_j A_1 + c\Lambda u_{ij} = 0, \quad (3.8b)$$

$$\Delta_Y A_1 + (D-2)u^{kl} \partial_k A_1 \partial_l A_1 + \left(\frac{2}{D-2} + c\right) \Lambda = 0, \quad (3.8c)$$

$$\varphi_c [\Delta_X A_0 + (n-2)q^{\rho\sigma} \partial_\rho A_0 \partial_\sigma A_0] = 0. \quad (3.8d)$$

Now we assume the metric in which

$$q_{\mu\nu}(X) dx^\mu dx^\nu = -dt^2 + a^2(t) \delta_{mn} dx^m dx^n, \quad (3.9a)$$

$$u_{ij}(Y) dy^i dy^j = dy^2 + \gamma_{ab}(Z) dz^a dz^b, \quad (3.9b)$$

where  $\gamma_{ab}(Z)$  is the metric of  $(D-n-1)$ -dimensional Einstein space. Let us consider the Eqs. (3.8b), (3.8c). If we set  $A_1 = A_1(y)$ , Eqs. (3.8b) and (3.8c) are reduced to

$$A_1 = k(y - y_0), \quad R_{ab}(Z) = -c\Lambda \gamma_{ab}(Z), \quad (3.10)$$

where  $k \equiv \pm\sqrt{-[c(D-2)+2]\Lambda/(D-2)}$ , and  $y_0$  is constant, and  $R_{ab}(Z)$  are the Ricci tensor of the metric  $\gamma_{ab}(Z)$ . The expression  $k$  gives  $\Lambda < 0$ .

Next we consider the Eqs. (3.8a) and (3.8d). Upon setting,  $A_0 = A_0(t)$ , the field equations Eq. (3.8a) give

$$(n-1) \left[ \left( \frac{\dot{a}}{a} \right) + \left( \frac{\dot{a}}{a} \right)^2 + \frac{\dot{a}}{a} \dot{A}_0 + \ddot{A}_0 \right] + \frac{\varphi_c^2}{2} \dot{A}_0^2 + c\Lambda e^{2A_0} = 0, \quad (3.11a)$$

$$\left[ \left( \frac{\dot{a}}{a} \right) + (n-1) \left( \frac{\dot{a}}{a} \right)^2 + (2n-3) \frac{\dot{a}}{a} \dot{A}_0 + \ddot{A}_0 + (n-2) \dot{A}_0^2 + c\Lambda e^{2A_0} \right] q_{mn} = 0, \quad (3.11b)$$

$$\varphi_c \left[ \ddot{A}_0 + (n-1) \frac{\dot{a}}{a} \dot{A}_0 + (n-2) \dot{A}_0^2 \right] = 0, \quad (3.11c)$$

where  $\dot{\phantom{x}}$  denotes the ordinary derivative with respect to the coordinate  $t$ . In the following, we look for the solution in the case  $c = 0$  or  $\varphi_c = 0$  because it is not so easy to find the solution analytically if both  $c$  and  $\varphi_c$  have non-zero values.

Let us first consider the case of  $c = 0$ , which corresponds to  $R_{ij}(Y) = 0$ . From the Eq. (3.11), we get

$$A_0(t) = \tilde{c}_1 t + \tilde{c}_3, \quad a(t) = \tilde{c}_4 e^{\tilde{c}_2 t}, \quad (3.12)$$

where  $\tilde{c}_i$  ( $i = 1, \dots, 4$ ) are constants, and  $\tilde{c}_1, \varphi_c$  are given by

$$\tilde{c}_1 = -\frac{n-1}{n-2} \tilde{c}_2, \quad \varphi_c^2 = \frac{2(n-2)}{n-1}. \quad (3.13)$$

If we define the cosmic time  $\tau$  by  $\tau = \tilde{c}_1^{-1} e^{\tilde{c}_1 t + \tilde{c}_3}$ , the scale factor of  $n$ -dimensional spacetime can be written as

$$a(\tau) = a_0 \tau^{1/(n-1)}, \quad (3.14)$$

where the constant  $a_0$  is given by  $a_0 = \tilde{c}_1^{1/(n-1)} \tilde{c}_4 e^{(n-2)\tilde{c}_3/(n-1)}$ . The  $D$ -dimensional metric thus becomes

$$ds^2 = e^{2A_1(y)} \left[ -d\tau^2 + \tau^{2/(n-1)} \delta_{mn} dx^m dx^n + dy^2 + \gamma_{ab}(Z) dz^a dz^b \right], \quad (3.15)$$

where we absorbed the inessential scaling into the metric. The solution implies that the power of the scale factor for  $n = 4$  is too small to give a realistic expansion law such as that in the matter dominated era ( $a \propto \tau^{2/3}$ ) or that in the radiation dominated era ( $a \propto \tau^{1/2}$ ).

Next we consider the case of  $\varphi_c = 0$ . Since the function  $A_0(x)$  again describes the conformal rescaling of the  $n$ -dimensional metric, the function  $A_0$  can take the constant value without loss of generality. Thus, from the Eq. (3.8a), we see that  $n$ -dimensional spacetime is an Einstein space. If  $c$  vanishes, we can no longer find any de Sitter compactifications. The contribution of the parameter  $c$  is actually

important and played a major role in our solution. The field equations Eq. (3.11) give

$$a(t) = \tilde{c}_0 e^{\pm \sqrt{-c\Lambda/(n-1)} t}, \quad (3.16)$$

where  $\tilde{c}_0$  is constant. Here we define the cosmic time  $\tau$  and Hubble parameter  $H$  as

$$H^2 = -\frac{c}{(n-1)}\Lambda. \quad (3.17)$$

Equation (3.17) denotes that  $R_{ij}(Y)$  should be positive so that  $H^2 > 0$  unless  $c\Lambda = 0$ . We choose the metric in which

$$q_{\mu\nu}(X)dx^\mu dx^\nu = -dt^2 + \tilde{c}_0^2 e^{2Ht} \delta_{mn} dx^m dx^n, \quad (3.18a)$$

$$u_{ij}(Y)dy^i dy^j = e^{-2A_1} dy^2 + E_s \gamma_{ab}(Z) dz^a dz^b, \quad (3.18b)$$

where  $E_s$  is constant, and  $\gamma_{ab}$  is the metric of  $(D-n-1)$ -dimensional Einstein space. Using the metric Eq. (3.18), the Einstein equations lead to

$$A_1(y) = \ln [\tilde{c}_1(y - y_0)], \quad E_s = \frac{D - n - 2}{(n-1)H^2}, \quad (3.19)$$

where  $y_0$  is constant, and  $\tilde{c}_1$  is given by

$$\tilde{c}_1^2 = \left[ 1 + \frac{2}{(D-2)c} \right] \frac{(n-1)H^2}{D-2}. \quad (3.20)$$

Hence the  $D$ -dimensional metric and the scalar field can be expressed as

$$ds^2 = [\tilde{c}_1(y - y_0)]^2 [-dt^2 + \tilde{c}_0^2 e^{2Ht} \delta_{mn} dx^m dx^n] + dy^2 + \frac{D - n - 2}{(n-1)H^2} [\tilde{c}_1(y - y_0)]^2 \gamma_{ab}(Z) dz^a dz^b, \quad (3.21a)$$

$$\phi(y) = -\frac{2}{\alpha_\phi} \ln [\tilde{c}_1(y - y_0)]. \quad (3.21b)$$

Note that there is a curvature singularity at  $y = y_0$ . If we choose  $c$  so that  $c = c_* := \frac{2(D-n-2)}{n(D-2)}$ , the internal space becomes a flat spacetime. Otherwise, although the internal manifold is topologically flat, it contains a deficit or surplus solid angle. A deficit solid angle is obtained for  $c > c_*$ , while a surplus one is done for  $c < c_*$ .

### 3.2 Compactifications with field strength

We now obtain the solution by generalizing the matter field beyond the scalar field system that we have considered so far. As an example, let us next consider the warped de Sitter compactification which include the metric  $g_{MN}$ , the scalar field  $\phi$ , and the field strength  $F$ .

The action we consider in the Einstein frame is given by

$$S = \frac{1}{2\kappa^2} \int \left[ \{R - 2e^{-\alpha\phi/(p-1)}\Lambda\} * \mathbf{1} - \frac{1}{2} d\phi \wedge * d\phi - \frac{1}{2 \cdot p!} e^{\alpha\phi} F \wedge * F \right], \quad (3.22)$$

where  $\kappa^2$  is the  $D$ -dimensional gravitational constant,  $*$  is the Hodge operator in the  $D$ -dimensional spacetime, and  $\phi$  is the scalar field, and  $F$  is  $p$ -form field strength, and  $\Lambda$ ,  $\alpha$  are constants.

The  $D$ -dimensional action Eq. (3.22) gives the field equations

$$R_{MN} = \frac{2}{D-2} e^{-\alpha\phi/(p-1)} \Lambda g_{MN} + \frac{1}{2} \partial_M \phi \partial_N \phi + \frac{1}{2 \cdot p!} e^{\alpha\phi} \left( p F_{MA_1 \dots A_{p-1}} F_N{}^{A_1 \dots A_{p-1}} - \frac{p-1}{D-2} g_{MN} F^2 \right), \quad (3.23a)$$

$$d * d\phi - \frac{\alpha}{2 \cdot p!} e^{\alpha\phi} F \wedge * F + \frac{2\alpha}{p-1} e^{-\alpha\phi/(p-1)} \Lambda * \mathbf{1} = 0, \quad (3.23b)$$

$$d(e^{\alpha\phi} * F) = 0. \quad (3.23c)$$

We adopt the following ansatz for the  $D$ -dimensional metric:

$$ds^2 = e^{2A(y)} [q_{\mu\nu}(X) dx^\mu dx^\nu + dy^2 + \gamma_{ab}(Z) dz^a dz^b], \quad (3.24)$$

where  $q_{\mu\nu}$  is the  $n$ -dimensional metric which depends only on the  $n$ -dimensional coordinates  $x^\mu$ , and  $\gamma_{ab}$  is the  $p(= D - n - 1)$ -dimensional metric which depends only on the  $p$ -dimensional coordinates  $z^a$ . As for the scalar field and the  $p$ -form field strength, we take

$$\phi = \frac{2}{\alpha} (p-1) A(y), \quad (3.25a)$$

$$F = f \Omega(Z), \quad (3.25b)$$

where  $f$  is a constant, and  $\Omega(Z)$  is the volume form of the  $Z$  space

$$\Omega(Z) = \sqrt{\gamma} dz^1 \wedge \dots \wedge dz^p. \quad (3.26)$$

Here  $\gamma$  denotes the determinant of the metric  $\gamma_{ab}(Z)$ .

Let us first consider the Einstein Eq. (3.23a). Using the assumptions (3.24) and (3.25), the Einstein equations Eq. (3.23a) are written by

$$R_{\mu\nu}(X) - q_{\mu\nu} \left[ A'' + (D-2) (A')^2 + \frac{2}{D-2} \Lambda - \frac{p-1}{2(D-2)} f^2 \right] = 0, \quad (3.27a)$$

$$(D-1)A'' + 2(p-1)^2 \alpha^{-2} (A')^2 + \frac{2}{D-2} \Lambda - \frac{p-1}{2(D-2)} f^2 = 0, \quad (3.27b)$$

$$R_{ab}(Z) - \gamma_{ab} \left[ A'' + (D-2) (A')^2 + \frac{2}{D-2} \Lambda + \frac{n}{2(D-2)} f^2 \right] = 0, \quad (3.27c)$$

where  $'$  denotes the ordinary derivative with respect to the coordinate  $y$ , and  $R_{\mu\nu}(X)$  and  $R_{ab}(Z)$  are the Ricci tensors of the metrics  $q_{\mu\nu}$  and  $\gamma_{ab}$ , respectively.

Let us next consider the gauge field. Under the assumption Eq. (3.23c), the Bianchi identity is automatically satisfied. Also the equation of motion for the gauge field becomes

$$d[f e^{(D-2)A}] \wedge \Omega(X) \wedge dy = 0, \quad (3.28)$$

where  $\Omega(X)$  is the volume form of the  $X$  space

$$\Omega(X) = \sqrt{-q} dx^0 \wedge \cdots \wedge dx^{n-1}. \quad (3.29)$$

Here  $q$  denotes the determinant of the metric  $q_{\mu\nu}(X)$ . Thus the gauge field equation is automatically satisfied under the assumption Eq. (3.25).

Now we consider the scalar field equation. Substituting the fields Eq. (3.25), and the metric Eq. (3.24) into the equation of motion for the scalar field Eq. (3.23c), we obtain

$$2(p-1)\alpha^{-1}e^{-2A} \left[ A'' + (D-2)(A')^2 - \frac{\alpha^2}{p-1} \left( -\frac{\Lambda}{p-1} + \frac{f^2}{4} \right) \right] = 0. \quad (3.30)$$

Combining Eqs. (3.27) and (3.30), the Einstein equations are conveniently equivalent to

$$R_{\mu\nu}(X) - \beta \left( -\frac{\Lambda}{p-1} + \frac{f^2}{4} \right) q_{\mu\nu}(X) = 0, \quad (3.31a)$$

$$(D-1)A'' + \frac{2(p-1)^2}{(D-2)\alpha^2} \left[ (D-2)(A')^2 - \frac{\alpha^2}{(p-1)} \left( -\frac{\Lambda}{p-1} + \frac{f^2}{4} \right) \right] = 0, \quad (3.31b)$$

$$R_{ab}(Z) - \left[ \beta \left( -\frac{\Lambda}{p-1} + \frac{f^2}{4} \right) + \frac{f^2}{2} \right] \gamma_{ab}(Z) = 0, \quad (3.31c)$$

where the constant  $\beta$  is defined by

$$\beta = \frac{\alpha^2}{p-1} - \frac{2(p-1)}{D-2}. \quad (3.32)$$

From Eqs.(3.30) and (3.31b), we get

$$A(y) = \ell (y - y_0), \quad (3.33)$$

where  $y_0$  is constant, and  $\ell$  is defined by

$$\ell = \pm \alpha \sqrt{\frac{1}{(p-1)(D-2)} \left( -\frac{\Lambda}{p-1} + \frac{f^2}{4} \right)}. \quad (3.34)$$

If we choose  $\alpha$  so that  $\beta > 0$ , it follows that

$$\alpha > \sqrt{\frac{2}{D-2}} (p-1), \quad \alpha < -\sqrt{\frac{2}{D-2}} (p-1). \quad (3.35)$$

Hence, the internal space  $Z$  has the positive curvature which is clear from an inspection of Eq. (3.31c). So the field equations lead to the  $D$ -dimensional metric

$$ds^2 = e^{2\ell(y-y_0)} \left[ -dt^2 + e^{2Ht} \delta_{mn} dx^m dx^n + dy^2 + \gamma_{ab}(Z) dz^a dz^b \right], \quad (3.36)$$

where the Hubble parameter  $H$  is given by

$$H^2 = \frac{\beta}{n-1} \left( -\frac{\Lambda}{p-1} + \frac{f^2}{4} \right). \quad (3.37)$$

In the limit of  $H \rightarrow 0$ , we see that the Ricci tensor of the  $Z$  space leads to  $R_{ab}(Z) \rightarrow \frac{1}{2}f^2\gamma_{ab}(Z)$ . As the internal space is essentially supported by the field strength, we can keep the geometrical property of the internal space in the limit  $H \rightarrow 0$ .

Before concluding this subsection, we comment about the ansatz for the field strength  $F$ . In this paper, we have simply assumed that the indices of non-vanishing components of the field strength could be along the internal space  $Z$  to obtain the de Sitter spacetime for warped compactifications. If the constant field strength  $F$  has the components along our  $(n+1)$ -dimensional spacetime  $M$ , the Ricci tensor on  $M$  is proportional to the  $(n+1)$ -dimensional metric with negative sign that is no longer de Sitter spacetime.

### 3.3 Brane world models

Now we construct cosmological models directly from our solution Eq. (3.36), following the brane world approach. Since the details are discussed in Ref. [49], we explain only the essence of our model. The  $D$ -dimensional metric Eq. (3.36) is given in terms of the warped product of the non-compact direction  $y$ , the  $(D-n-1)$ -dimensional compact space  $Z$  and the  $n$ -dimensional de Sitter spacetime  $X$ . In Ref. [49], we fix  $n=4$ ). To realize the finite bulk volume, we cut the spacetime along  $y=y_0$ , where  $y_0$  is a constant. Then, the original spacetime is glued to its identical copy across the codimension-one discontinuity at  $y=y_0$ , which is now identified as the  $(D-1)$ -dimensional brane world composed of the  $X$  and  $Z$  spaces. Although our construction is very similar to the five-dimensional Randall-Sundrum (RS) model [48], a manifest difference is that our brane world still contains the compact internal space  $Z$ . Thus after integrating over  $Z$  on the brane, we can identify the  $n$ -dimensional de Sitter space  $X$  as our  $n$ -dimensional Universe with a finite effective 4D Planck mass. It also should be noted that the inclusion of the gauge field strength acting on the  $Z$  space is necessary to obtain a finite size of the compact internal space in the limit of the vanishing de Sitter expansion rate, which is the realization of the modulus stabilization of the  $Z$  space in the brane world.

We then discuss the spectrum of the tensor metric perturbations realized in our four-dimensional de Sitter universe. In Ref. [49], we showed the existence of the three kinds of eigen modes realized in our Universe, i.e., the massless zero mode, the continuum of Kaluza-Klein (KK) modes associated with the noncompact direction  $y$  and the discrete KK modes arising due to compactification of  $Z$  on the brane. The zero mode would reproduce the 4D gravity on the brane, and all the KK excitations are heavier than the critical mass in the de Sitter space, above which the wavefunction of the corresponding mode exhibits a damped oscillation. Thus our four-dimensional

de Sitter Universe is free from any massive excitation [49]. We also showed that both the original model found in [47] and our model suffer an instability against the scalar perturbations, associated with the decompactification in the  $y$  direction, although in our model the internal space  $Z$  can be fixed by the field strength. We expect that Casimir effects of the bulk fields would stabilize the  $y$  space, and this subject is under active study [55].

In the rest of the paper, we will present another construction of the cosmological models based on the lower-dimensional effective theory, integrating over the  $Z$  space. In the second approach, the noncompact  $y$ -direction should be regarded as an external direction in our Universe in contrast to the brane world approach.

#### 4. Lower-dimensional effective theory

In this section, we will consider another construction of our de Sitter Universe as well as fixing the moduli in the lower-dimensional effective theory, following the ordinary compactification approach. For simplicity, we consider the internal moduli degrees of freedom of the metric of internal space  $Z$  in the present paper.

Now, we assume the  $D$ -dimensional metric

$$ds^2 = e^{2A(v)} [w_{PQ}(M) dv^P dv^Q + e^{2\psi(v)} \gamma_{ab}(Z) dz^a dz^b] , \quad (4.1)$$

where  $\psi(v)$  is the moduli degrees of freedom on the  $Z$  space, and  $w_{PQ}(M)$  is the  $(n+1)$ -dimensional metric, and  $\gamma_{ab}(Z)$  is the metric of the  $(D-n-1)$ -dimensional Einstein space  $Z$ .

In contrast to the previous section,  $M$  contains the direction of the infinite line  $y$  in the previous sections, which is regarded as an external direction in the effective theory approach. Furthermore, we assume that the scalar fields are expressed as in Eq. (3.25a), and  $F$  is a fixed by Eq. (3.25b). Hence, the moduli  $\psi$  and the function  $A$  are the only dynamical variables in the effective theory. In the following, we construct the  $(n+1)$ -dimensional effective theory after compactifying the  $Z$  space. However, in contrast to the brane world model constructed from the higher-dimensional solution Eq. (3.36) where the  $n$ -dimensional spacetime  $X$  becomes our external directions, now the  $(n+1)$ -dimensional spacetime  $M$  including the noncompact direction  $y$  is assumed to be our external directions. The  $(n+1)$ -dimensional effective action for these variables can be obtained by evaluating the  $D$ -dimensional action Eq. (3.22). First, for the metric Eq. (4.1), the  $D$ -dimensional scalar curvature  $R$  is expressed as

$$R = e^{-2A} [R(M) + e^{-2\psi} R(Z) - 2(n-1)\Delta_M A + 2p\Delta_M \psi + p(p+1)w^{PQ}\partial_P \psi \partial_Q \psi + \{(n-1)(p-1) + p(D-1)\} w^{PQ}\partial_P A \partial_Q A + 2p(D-1)w^{PQ}\partial_P A \partial_Q \psi] , \quad (4.2)$$

where  $\Delta_M$  and  $\Delta_Z$  are the Laplace operators on the space of  $M$  and the space  $Z$ , and  $R_{PQ}(M)$  and  $R_{ab}(Z)$  are the Ricci tensors of the metrics  $w_{PQ}$  and  $\gamma_{ab}$ , respectively.



We assume that the internal space  $Z$  satisfies the condition  $R_{ab}(Z) = \lambda \gamma_{ab}(Z)$ , where  $\lambda$  characterizes the internal space curvature. Here  $\beta$  is given by (3.32). Inserting the Eqs. (3.25) and (4.2) into Eq. (3.22), we get

$$S = \frac{1}{2\tilde{\kappa}^2} \int_M e^{(D-2)A+p\psi} \left[ \left\{ R(M) + p\lambda e^{-2\psi} - 2\Lambda - \frac{f^2}{2} \right\} *_M \mathbf{1}_M - 2(n-1)d *_M dA - 2pd *_M d\psi - \{(n-1)(p-1) + p(D-1)\} dA \wedge *_M dA - 2p(D-1)dA \wedge *_M d\psi - p(p+1)d\psi \wedge *_M d\psi \right], \quad (4.3)$$

where  $*_M$  is the Hodge operator on the  $M$  space and  $\tilde{\kappa}$  is given by  $\tilde{\kappa} = V^{-1/2}\kappa$  with the volume of the internal space  $Z$

$$V \equiv \int_Z *_Z \mathbf{1}_Z. \quad (4.4)$$

Here  $*_Z$  is the Hodge operator on the  $Z$  space.

Using the conformal transformation  $w_{PQ}(M) = e^{-2[(D-2)A+p\psi]/(n-1)} w_{PQ}(\bar{M})$ , the  $(n+1)$ -dimensional action Eq. (4.3) is expressed in terms of the variables in the Einstein frame as

$$S = \frac{1}{2\tilde{\kappa}^2} \int_{\bar{M}} \left[ \{ R(\bar{M}) - V(\bar{A}, \bar{\psi}) \} *_M \mathbf{1}_{\bar{M}} - \frac{1}{2} d\bar{A} \wedge *_M d\bar{A} - \frac{1}{2} \frac{c_2}{\sqrt{c_1 c_3}} d\bar{A} \wedge *_M d\bar{\psi} - \frac{1}{2} d\bar{\psi} \wedge *_M d\bar{\psi} \right], \quad (4.5)$$

where  $R(\bar{M})$  is the Ricci scalar with respect to the metric  $w_{PQ}(\bar{M})$ , and we have dropped the surface terms coming from  $\Delta_{\bar{M}} A$ ,  $\Delta_{\bar{M}} \psi$ , and the potential  $V(\bar{A}, \bar{\psi})$ , fields  $\bar{A}$ ,  $\bar{\psi}$ , the constants  $c_i$  ( $i = 1, 2, 3$ ) are defined by

$$V(\bar{A}, \bar{\psi}) = \exp \left[ -\frac{2(D-2)\bar{A}}{(n-1)\sqrt{c_1}} \right] \left[ 2\Lambda \exp \left\{ -\frac{2p\bar{\psi}}{(n-1)\sqrt{c_3}} \right\} + \frac{f^2}{2} \exp \left\{ -\frac{2np\bar{\psi}}{(n-1)\sqrt{c_3}} \right\} - p\lambda \exp \left\{ -\frac{2(D-2)\bar{\psi}}{(n-1)\sqrt{c_3}} \right\} \right], \quad (4.6a)$$

$$\bar{A} = \sqrt{c_1} A, \quad \bar{\psi} = \sqrt{c_3} \psi, \quad (4.6b)$$

$$c_1 = 2 \left[ \frac{n}{n-1} (D-2) - 2(D-1) \right] (D-2) + 2 \left[ n-1 + \frac{2}{\alpha^2} (p-1) \right] (p-1) + 2p(D-1), \quad (4.6c)$$

$$c_2 = \frac{4(D-2)p}{n-1}, \quad (4.6d)$$

$$c_3 = 2p \left( \frac{n-1}{p} + 1 \right). \quad (4.6e)$$

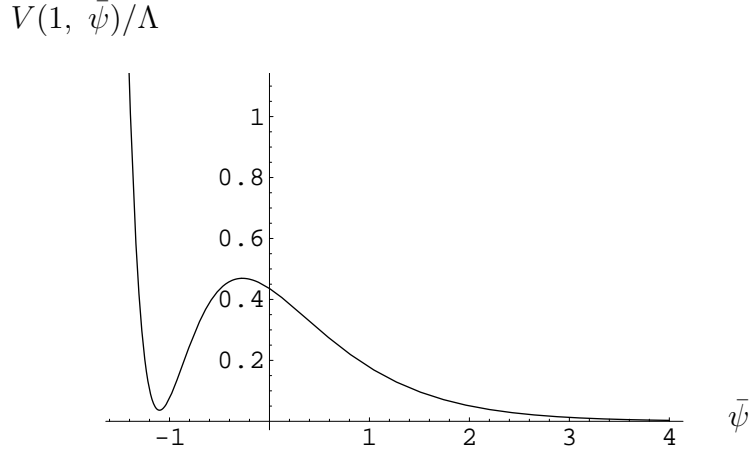
Here  $\Delta_{\bar{M}}$  is the Laplace operator constructed from the metric  $w_{PQ}(\bar{M})$ . The form of the potential Eq. (4.6a) implies that the warp factor  $\bar{A}$  cannot be fixed by the

background fields. This is consistent with the fact that the analysis of the the scalar perturbations for the solutions which are discussed in this paper is unstable. We have explained these in [49].

Then, in the following, we fix the value of  $\bar{A}$  by assuming some additional stabilization mechanism which does not affect the dynamics of the other moduli  $\bar{\psi}$ , and consider the stabilization of  $\bar{\psi}$ . In order to fix the moduli degrees of freedom  $\bar{\psi}$ , the moduli potential has to have a minimum or at least a local minimum. The moduli potential energy at the minimum is given by

$$V(\bar{A}, \bar{\psi}_0) = \exp \left[ -\frac{2(D-2)\bar{A}}{(n-1)\sqrt{c_1}} \right] \left[ -\frac{1}{2}(n-1)f^2 \exp \left\{ -\frac{2pn\bar{\psi}_0}{(n-1)\sqrt{c_3}} \right\} - \lambda(D-p-2) \exp \left\{ -\frac{2(D-2)\bar{\psi}_0}{(n-1)\sqrt{c_3}} \right\} \right]. \quad (4.7)$$

In the present model, setting  $n = 3$ ,  $p = 6$ ,  $\bar{A} = 1$ ,  $f = 0.17$ ,  $\lambda = 0.229$  and  $\alpha = 0.5$  in the unit of  $\Lambda = 1$ , we find the stable minimum point  $\bar{\psi}_0 = -1.1000$ , where  $\Lambda^{-1}V(1, \bar{\psi}_0) = 0.0365$ . Since the potential energy is proportional to the warp factor  $e^{-\bar{A}}$ , the  $(n+1)$ -dimensional effective cosmological constant can be dropped exponentially as  $\bar{A}$  increases. Hence, if we choose the value of the function  $\bar{A}$  appropriately, we get the small energy value of the potential at  $\bar{\psi} = \bar{\psi}_0$ . We illustrate the moduli potential in Fig. 1 and 2.

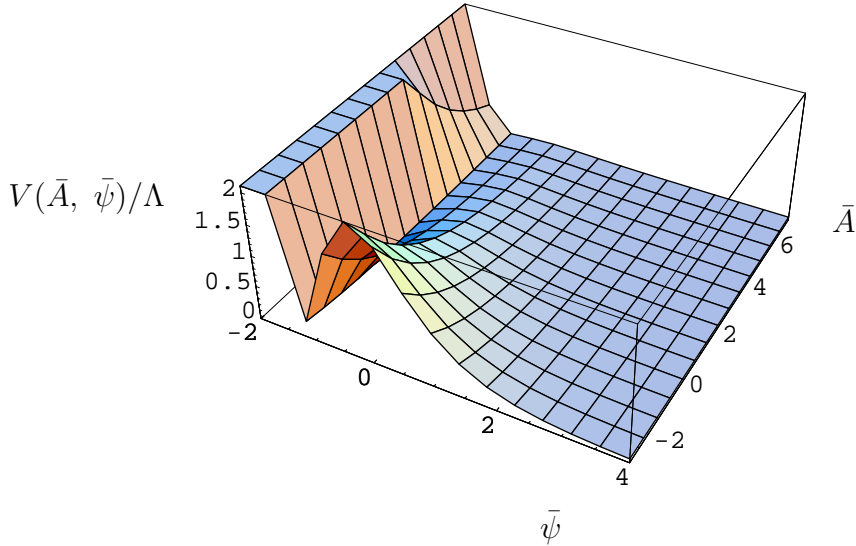


**Figure 1:** The moduli potential given in (4.6a) is depicted. We set  $n = 3$ ,  $p = 6$ ,  $\bar{A} = 1$ ,  $f = 0.17$ ,  $\lambda = 0.229$  and  $\alpha = 0.5$  in the unit of  $\Lambda = 1$ . The minimum of the potential is located at  $\bar{\psi}_0 = -1.1000$  and its value is  $\Lambda^{-1}V(1, \bar{\psi}_0) = 0.0365$ . If we fix the value of the function  $\bar{A}$ , we get the small energy value of the moduli potential at the local minimum.

In order to give the positive value of the potential energy, the constant  $\bar{\psi}_0$  should satisfy the condition

$$\exp \left[ -\frac{pn\bar{\psi}_0}{(n-1)(D-2)\sqrt{c_3}} \right] < \frac{2}{(n-1)f^2} \lambda(D-p-2). \quad (4.8)$$

In the gravity system with a positive cosmological constant and finite volume of the internal space, upon setting  $D > p + 2$ , a positive curvature term of the internal space gives a dominant contribution to the moduli potential at small  $\bar{\psi}$ , while a positive cosmological constant term becomes dominant for large  $\bar{\psi}$ . Hence the moduli potential is unbounded from below as  $\bar{\psi} \rightarrow -\infty$  and drops exponentially as  $\bar{\psi} \rightarrow \infty$ . Then the internal space  $Z$  either shrinks to zero volume or is decompactified. We also obtain the run away type moduli potential even if  $\Lambda < 0$ . However, if we include the field strength with a cosmological constant, we can find a stable minimum for the moduli potential. The cosmological constant and the field strength force to expand the extra dimension while the curvature of the internal spacetime forces to contract it. These combinations produce a local minimum of the effective potential. Hence the role of the field strength is important to find a minimum of the potential [50]. The potential energy at the minimum is equivalent to the  $(n + 1)$ -dimensional cosmological constant. Then we can obtain the stable in the  $dS_{n+1} \times Z$  background. If the universe may be created on the top of the potential hill, the universe rolls down to the potential minimum whose value is positive. For other choice of the parameters, we can also have a negative potential minimum. Then the universe evolves into the static  $AdS_{n+1} \times Z$ , which is a stable spacetime. As the moduli potential without the cosmological constant cannot provide the positive value at the local minimum even if we have a field strength, the contribution of the cosmological constant is essential to derive de Sitter compactifications.



**Figure 2:** The moduli potential given in (4.6a) is depicted. We set  $n = 3$ ,  $p = 6$ ,  $f = 0.17$ ,  $\lambda = 0.229$  and  $\alpha = 0.5$  in the unit of  $\Lambda = 1$ . We can find the minimum of the potential with respect to the field  $\bar{\psi}$  while there is not local minimum of the potential for the direction of  $\bar{A}$ .

## 5. Discussions

In the first part of this paper, we have discussed the warped de Sitter compactifications in higher-dimensional theory. We have obtained warped de Sitter spacetimes with a positively or negatively curved internal space to investigate the possible generalizations of the solutions found in [47]. We also presented the  $n$ -dimensional warped de Sitter compactification due to the contribution of the matter fields. These solutions give the accelerating expansion of the  $n$ -dimensional universe. The construction of the cosmological model is directly obtained from our solutions, following the brane world approach (see [49] for details). The solutions without field strength were obtained by the product type form of the warp factor. This structure requires that warp factor of the metric is expressed as the form  $A_0(x)A_1(y)$ , where  $A_0(x)$  is a function of the four-dimensional coordinates  $x^\mu$ , and  $A_1(y)$  is a function on the internal space. In the pure gravity system, as the function  $A_0(x)$  in the metric describes the conformal rescaling of the  $n$ -dimensional metric, we could set  $A_0 = 0$  without loss of generality. Supposing that our universe stays at a constant position in the internal space  $Y$ , we have shown that the evolution of  $n$ -dimensional universe gives an  $n$ -dimensional de Sitter space or FRW universe. The solutions in the  $D$ -dimensional theory give the  $n$ -dimensional de Sitter spacetime for the case of not only the pure gravity system field but also the gravity with matter fields which depend only on the coordinates of  $y$ . However, if the scalar field depends only on the coordinate of the  $n$ -dimensional spacetime, the solution of field equations denotes that the power of the scale factor of  $n$ -dimensional spacetime is too small to give a realistic expansion law for  $n$ -dimensional FRW universe. For the compactification with field strength, we have simply required that non-vanishing components of the field strength should be along the internal space  $Z$ . An important consequence of this is obtaining the de Sitter spacetime from warped compactifications. If the constant field strength  $F$  goes through our  $n$ -dimensional spacetime  $X$ , the Ricci tensor on  $X$  is proportional to the  $n$ -dimensional metric with negative sign that is no longer de Sitter spacetime. Hence we were only concerned with the field strength along  $Z$ .

In the second part of this paper, we have presented another construction of a de Sitter spacetime in terms of the  $(n+1)$ -dimensional effective theory after integrating over the  $Z$  space. The  $(n+1)$ -dimensional spacetime  $\bar{M}$  was regarded as our Universe. We have calculated the moduli potential and discussed its stability using the moduli potential. Assuming some stabilization mechanism of the volume moduli which does not affect the dynamics of the internal space moduli (for example, via Casimir effects of the bulk fields [55]), the cosmological constant and field strengths force the internal space to expand, while the curvature of the internal spacetime forces it to contract. These combination produces a local minimum of the moduli potential. In the  $(n+1)$ -dimensional effective theory, the scale of the compact internal space  $Z$  is stabilized by balancing the gauge field strength wrapped around the internal space and the

curvature term of the internal space with the cosmological constant. For some choices of the parameters, these contributions could produce a local minimum of the effective potential with a positive value which corresponds to the  $dS_{n+1} \times Z$  background in the effective theory, while the solution in the original higher-dimensional gravitational theory gave the geometry of  $dS_n \times \mathbb{R} \times Z$ . If the  $(n+1)$ -dimensional universe is created near the top of the potential hill, the moduli rolls down the potential hill and finds a stable minimum point. Since the moduli potential eventually turns out to be positive or negative, the  $(n+1)$ -dimensional background geometry becomes  $dS_{n+1}$  or  $AdS_{n+1}$  spacetime. As the moduli potential without the cosmological constant leads to the  $AdS_{n+1}$  at the local minimum even if we have a field strength, we need the contribution of the cosmological constant to obtain de Sitter compactifications.

The next step may be to develop a framework to understand generalizations of warped compactifications in which one varies the matter fields or the boundary conditions or other details though this program has not been actually achieved in the present paper. Along this way, we will clarify some properties of cosmic acceleration for the warped compactification elsewhere.

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